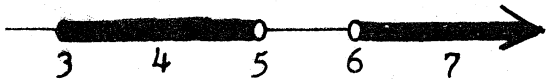
January 9, 1980 WOCOMAL FRESHMAN MEET

ROUND I: GRAPHING ON THE NUMBER LINE

ON THE NUMBER LINES BELOW DRAW THE GRAPHS OF THE SOLUTIONS OVER THE SET OF REAL NUMBERS (UNLESS INDICATED OTHERWISE) FOR THE FOLLOWING OPEN SENTENCES. USE THIS NOTATION FOR $3 \leq x < 5$ or $x > 6$.

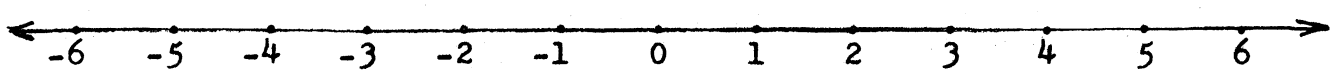


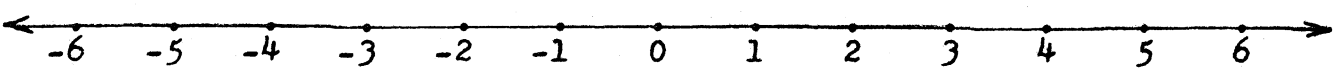
(1 point) 1. $0 < x + 2 \leq 5$.

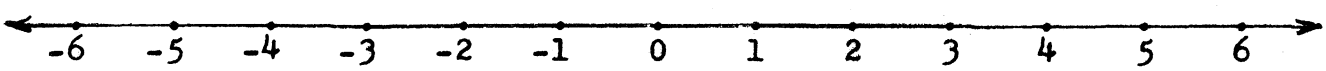
(2 points) 2. $\{x: \ 3 - 2x \leq 6 \ \text{ or } \ 5 - 2x \geq 9, \ x \text{ is an integer}\}$.

(3 points) 3. $1 \leq |2 - x| < 3$.

ANSWERS:

1. 

2. 

3. 

St. John's, Southbridge, Tantasqua

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ROUND II: SYMMETRY, TRANSLATION, ROTATION, REFLECTION IN COORDINATE PLANE

ALL ANSWERS MUST BE EXPRESSED AS ORDERED PAIRS OF NUMBERS REDUCED TO SIMPLEST FORM.

1. Find the image of point $A(2,2)$ under a reflection in the x-axis followed by a reflection in the line $x = 3.5$.

2. Give the new coordinates of the vertices of a triangle if it is rotated 90° counterclockwise and the original coordinates are $(1,-4)$, $(3,1)$, $(7,-1)$.

3. Given that three of the vertices of a rhombus are $(-2,2)$, $(-2,-3)$, and $(1,1)$. Find the coordinates of the point of symmetry of the rhombus. (A rhombus is a parallelogram with four sides of equal length. )

ANSWERS: (1 point) 1. (,)

(2 points) 2. (,), (,), (,)

(3 points) 3. (,)

Bromfield, quaboag

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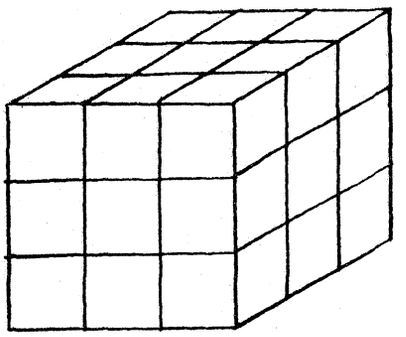
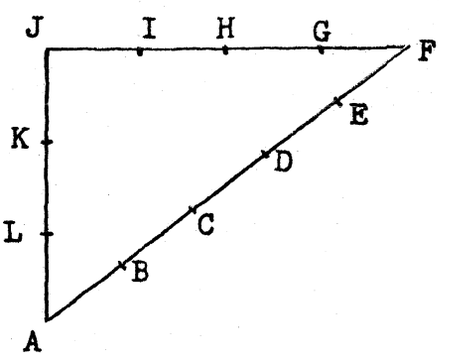
ROUND III: OPEN

1. A solid white cube is painted red on the outside of all 6 faces. If the cube is cut into 27 equal small cubes, how many have exactly two red faces?



2. If $6! = 6x5x4x3x2x1$ reduce the fraction $\frac{14!}{16! - 15!}$ to the simplest possible fractional form.

3. This triangle AFJ has sides of length 3, 4 and 5. Every unit point along the sides is marked with one of the letters A to L. An elf has 200 units of spider webbing. If he begins to wrap it around the triangle starting at point A, name the letter of the point at which he finishes.



ANSWERS: (2 points) 1. _____

(2 points) 2. _____

(2 points) 3. _____

Shrewsbury, Southbridge, Worcester Academy

January 9, 1980 WOCOMAL FRESHMAN MEET

ROUND IV: OPERATIONS ON NUMERICAL FRACTIONS & DECIMALS

EXPRESS EACH OF THE FOLLOWING AS A FRACTION REDUCED TO SIMPLEST FORM.

1. $\dfrac{\frac{3}{8} - \frac{1}{12}}{\frac{6}{5} + \frac{3}{10}} =$

2. $[(14.21 \div 3.5) + 0.8] \cdot 0.2 =$

3. $0.\overline{12} - 0.0\overline{12} =$

ANSWERS: (1 point) 1. _____

(2 points) 2. _____

(3 points) 3. _____

Algonquin, Quaboag, South

January 9, 1980 WOCOMAL FRESHMAN MEET

TEAM ROUND: NUMBER THEORY, PRIMES, DIVISIBILITY, LCM, GCF, SEQUENCES

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST FORM. EACH ANSWER TWO POINTS.

1. Determine the next number in the sequence 72, 63, 55, 48, 42, ___. 1. _____

2. Which of the following numbers are divisible by 3, 4 and 5? (a) 8361984000 (c) 7832948640 (b) 1648038425 (d) 5419275080 2. _____

3. What is the ratio of the least common multiple of 3, 4, 5 to the greatest common factor of 28, 42, 56? 3. _____

4. What is the least common multiple of 84 and 990? 4. _____

5. Find the greatest common factor of 504 and 945. 5. _____

6. How many ordered triples of positive integers (x,y,z) are there if the greatest common factor of (x,y,z) is 20 and the least common multiple of (x,y,z) is 60? 6. _____

7. Find the sum of the three largest prime factors of 18018. 7. _____

8. Find the greatest common factor of the product and the sum of the greatest common factor and the least common multiple of 20 and 15. 8. _____

9. Find the sum of all three-digit counting numbers the sum of the squares of whose digits is 30. 9. _____

10. A sequence is formed by starting with any integer and then squaring its digits and adding the squares to form the next number of the sequence. "Happy numbers" are numbers that follow this sequence and become 1. (Example: 32, 9+4=13, 1+9=10, 1+0=1) Which of the following numbers are happy? (a) 10,000 (b) 31 (c) 87 (d) 23 10. _____

11. If the difference between two primes is 2, such primes are said to be twin primes. Find the sum of the greatest twin primes and the smallest twin primes less than 100. 11. _____

12. Ray and Tom want to go to a hockey game together. Ray is off work every ninth day. Tom is off every seventh day. If Ray is off work today and Tom is off work tomorrow, how many days after today will they both be off work on the same day so that they can go to the game together? 12. _____

Auburn, Hudson, Hudson Catholic, Marlboro, St. John's, Shepherd Hill, Shrewsbury, Southbridge, Worcester Academy

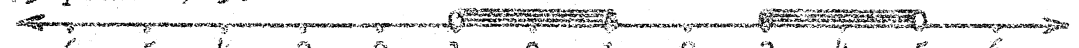
January 9, 1980 wocomal freshman meet answers

ROUND I

(1 point) 1.

(2 points) 2.

(3 points) 3.



ROUND II

(1 point) 1. (5, -2)

(2 points) 2. (4, 1), (-1, 3), (1, 7)

(3 points) 3. $(-\frac{1}{2}, \frac{3}{2})$ or (-0.5, 1.5)

ROUND III

(2 points) 1. 12

(2 points) 2. $\frac{1}{225}$

(2 points) 3. E or I

ROUND IV

(1 point) 1. 7/36

(2 points) 2. 243/250

(3 points) 3. 6/55

TEAM ROUND

(2 points each)

1. 37
2. a, c
3. 30/7
4. 13860
5. 63
6. 6
7. 31
8. 5
9. 1776
10. a, b, d
11. 152
12. 36